LAIKIPIA UNIVERSITY JOURNAL OF SCIENCE AND APPLIED TECHNOLOGY

# Optimizing a Hybrid Round-Bottom Triangular Open-Channel For Storms

**John Wahome**

Department of Mathematics, Laikipia University, Kenya

**Corresponding author email:** jwahome@laikipia.ac.ke

## Abstract

According to the Fluid Flow Equation, the mass flow rate of a fluid is the product of its density, area and velocity. Fast flowing storm water could therefore cause a sudden increase in fluid velocity and flooding, an increasingly common challenge as the effects of global warming become more pronounced. These might overshoot certain desirable thresholds and damage the channel or canal, by scouring. Similarly, the Continuity Equation guarantees that the velocity of a fluid decreases the closer a location is from the bottom. This implies a converse danger of siltation when the speed of flow is too sluggish. For that reason, channel designers carefully choose shapes with dimensions which maximize discharge, while keeping siltation in check. They also seek to slow down the velocity of the channel's flow by making it dissipate much of its load in case of an overflow. This can be partially achieved by an appropriate design of the area above the channel. Meta-heuristic, nondominated sorting genetic algorithms, ant-colony optimisation, differential evolution algorithm (DEA), sequential quadratic programming (SQP) and Lagrange multipliers are some of the methods deployed in minimising the cost function subject to the cross-section of a channel. In practice, channel design hydrodynamics and engineering will involve more parameters than those that this paper covers, including the type of construction materials used to line the channel. Several studies have shown that for a given discharge value and for all slopes, the total cost of construction of a compound triangular cross-section with a rounded bottom is always less than the cost of trapezoidal cross-sections. This paper assumes other factors optimum and applies a purely mathematical approach to determine the best round bottomed triangular open channel design which additionally decreases velocity fluctuations during storms.

**Keywords:** Discharge, hydraulic radius, Manning equation, open channel, wetted perimeter



**Received: 20-06-2023** **Accepted: 20-09-2023** **Published: 27-12-2023**

## Introduction

The main difference between fluid flowing in a closed pipe (*full-bore*) and the other type of flow categorised as *open-channel* is that the latter has a free and exposed surface that is subject to atmospheric pressure, while in the former, the flow is determined entirely by solid boundaries. Wahome (2014) gives a more detailed comparison of the two flows. Familiar open-channels flows include rivers and streams which are *natural*, while canals, flumes, spillways are examples of *artificial* channels.

The *hydraulic efficiency* of a channel depends on its shape. Therefore, the channel shape which provides maximum discharge for a fixed bed slope, roughnes and fixed area is the most efficient. According to Massey and Smith (2006), the formulae of Manning and Chezy among others predict that for a uniform flow with a given bed gradient, the hydraulic mean depth $m$ affects the factors which influence channel efficiency such as discharge ($Q$), mean velocity, roughness and cross-sectional area. The hydraulic mean depth, $m$, is defined as the ratio of the flow area $A$ to the wetted perimeter, $P$. The less the wetted perimeter, the greater the $m$ and the discharge, and the less the cost of lining materials.

It has been established in open-channel flow studies that the semi-circular bottom gives the maximum hydraulic mean depth according to Douglas et al. (2001), but in practice this shape is useful only for small channels since some other factors such as the need for a reasonable angle of repose for granular banking material, relative ease of construction and cost excavation often override. Trapezoidal channels, which include the triangular and rectangular shapes, are more widely used. These are the ones encountered in practice, especially where the digging is done manually by shovels. For any shape adopted for a channel, different bases and angles will give different efficiencies, so that there is a particular configuration which gives the most discharge per a certain amount of excavation.

This paper assumes the fluid flow which is irrotational, inviscid, steady and incompressible neglets surface tension and viscosity, and considers only mathematical hydraulic efficiency to highlight the most economical dimensional characteristics that minimise velocity flunctuations, scouring and siltation for the round-bottomed triangular open channel. An appropriate extension of the section above the free surface to ameliorate the effects of overflow due to storms is also proposed.


**Received: 20-06-2023 Accepted: 20-09-2023 Published: 27-12-2023**

Volume.1, Number. 1, 2023 ISSN 22791

**Mathematical Principles**

**Application of Manning's Equation in Treatment of Common Channel Shapes**

In the context of open channel flow, Marriott and Uddin (2009) treat the terms *best hydraulic section* and *most economic section* as synonymous. The more a channel gives *the maximum discharge for a given amount of excavation*, the more economical it is said to be (Rajput, 2006). At the heart of most open channel flow computations lies the Manning's equation,

$$\boldsymbol{Q} = \frac{\sqrt{(S_0)}\; m^{\frac{2}{3}} A}{n} \qquad (2.1)$$

in which $\mathbf{Q}$ represents the flow rate in the channel, $\mathbf{S_0}$ the slope of the channel, A the cross-sectional area of the channel and *m* is the hydraulic radius of the channel, defined as the ratio of cross-sectional area to the wetted perimeter, P. The equation (2.1) above may also be reformulated as,

$$\boldsymbol{A} = (\boldsymbol{P})^{\frac{2}{5}} \left(\frac{n\,Q}{\sqrt{S_0}}\right)^{\frac{3}{5}} \qquad (2.2)$$

The equation (2.2) demonstrates that if $S_0$, $\mathbf{Q}$ and n held constantly, A and P are directly proportional, and that gives a way of minimising the flow area (and thus maximising the efficiency of the channel).

In a way, the round-cornered triangular channel is a special case of a simple trapezoidal channel which is the reason this paper uses the latter to demonstrate the application of Manning's equation in determining the optimum dimensions of a channel.

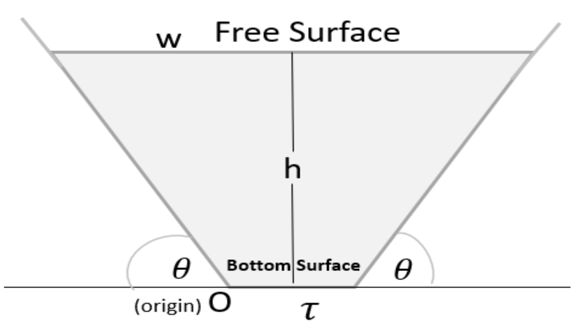


**Fig. 1: The Trapezoidal Open Channel**


Received: 20-06-2023 Accepted: 20-09-2023 Published: 27-12-2023

Volume.1, Number. 1, 2023 ISSN 22791

The wetted area and wetted perimeter of the trapezoidal channel illustrated in the Figure 1 are,

$$\boldsymbol{A = \int_0^h \int_{ycot(\pi-\theta)}^{ycot(\theta)+\tau} dxdy = \tau\, h + h^2\, cot\theta} \quad (2.3)$$

$$\boldsymbol{P = \tau + 2\, h\, cosec\theta} \quad (2.4)$$

$$\boldsymbol{m = \frac{A}{P} = \frac{A}{\tau+2\, h\, cosec\theta} = \frac{A}{\frac{A}{h} - h\, cot\theta + 2h\, cosec\,\theta},\ \left(since\ \tau = \frac{A}{h} - \ h\, cot\theta\right)} \quad (2.5)$$

This means that $m = \frac{A}{\frac{A}{h} - \ h\, cot\theta + 2h\, cosec\,\theta}$, which is greatest when the denominator is minimised with respect to $h$. Furthermore,

$$\frac{\partial}{\partial h}\left(\frac{A}{h} \ - \ \ h\, cot\theta + 2h\, cosec\,\theta\right) = -h(-cosec^2\theta) + 2h(-cosec\theta cot\theta) = 0 \quad (2.6)$$

Thus,

$$\frac{1}{sin\theta}(1 - 2cos\theta) = 0 \ \Longrightarrow \theta = \frac{\pi}{3} \quad (2.7)$$

This means that the trapezoidal channel is most efficient when $\theta = \frac{\pi}{3}$ , $i.e.\ 60^0$, which is the half of a hexagon.

Additionally, with $\theta = \frac{\pi}{3}$, we also infer a relationship between the sides; i.e.,

$$\tau = \frac{2}{\sqrt{3}}\, h \quad (2.8)$$

The other relationships such as wetted perimeter, *P,* and cross-sectional area, *A,* for the trapezoidal channel are similarly derived; that is,

$$P = h\, 2\sqrt{3}, A = h^2\,\sqrt{3}, P = 3\tau, \textit{surface width}\, w = \frac{4}{\sqrt{3}}\, h \textit{ and bottom-width to depth ratio, } \frac{\tau}{h} = 2(cosec^2\theta - \ cot\theta) \quad (2.9)$$

Table 1and Figure 2 compare the efficiencies of channels having number of sides near the six of the hexagonal one. It was generated using Microsoft Excel.


**Received: 20-06-2023 Accepted: 20-09-2023 Published: 27-12-2023**

**Table 1: Efficiency Values for Polygons Around the Hexagon with A = 100 and η = 50**

| Trapezoidal Channel Slope (radians) | No. of sides | Wetted Perimeter | Efficiency |
|---|---|---|---|
| 1.571 | 4 | 102 | 0.01 |
| 1.257 | 5 | 90.9 | 0.011 |
| 1.0472 | 6 | 88.6 | 0.01129 |
| 0.897598 | 7 | 90.03112933 | 0.011107 |
| 0.78539825 | 8 | 93.42135265 | 0.010704 |

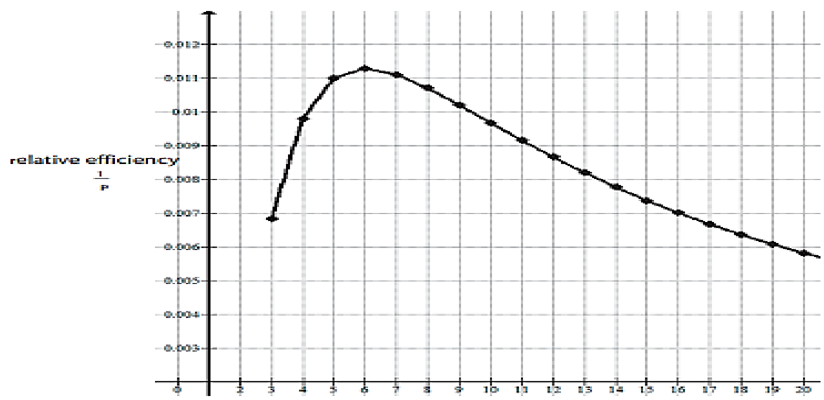


**Fig. 2: The relative efficiencies of channels near the hexagonal one, with A = 100 and h = 50**

The graph peaks at $n = 6$ therefore supporting the result that the hexagon is the most efficient trapezoidal design. It also reveals that a 7-sided (heptagonal) cross-section at 0.011107269 is more efficient than a 5-sided (pentagonal) one at 0.011001071.

## The Hybrid Triangular Channel With Rounded Bottom

Although the semi-circular channel is the most efficient, it has limited practical usability. This creates need for hybrid channels which are trapezoidal but with rounded bottoms. Froehlich (2008), Chahar and Basu (2009), Hameed (2010) and several others have applied different techniques to specify the optimum dimensions of various types of hybrid round-bottom channels. This paper has focused on the round-bottomed channel shown in Figure 3, which will be treated as a special case of the trapezoidal one with round corners.


Received: 20-06-2023 Accepted: 20-09-2023 Published: 27-12-2023

Volume.1, Number. 1, 2023 ISSN 22791

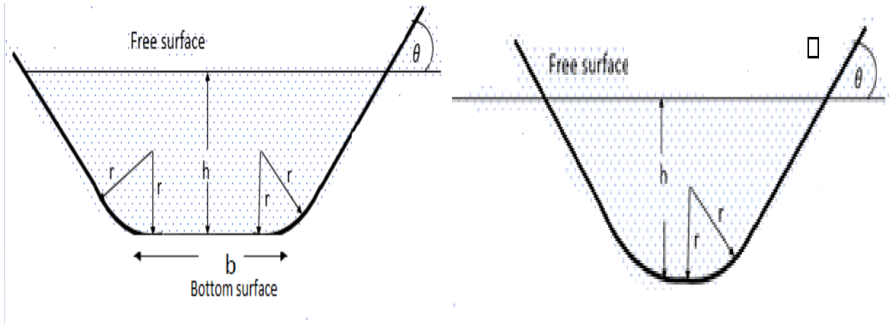


**Fig. 3: A rou**

In Figure 3, $r$ is a scalar multiple of h, so that $r = \zeta h, \quad 0 \leq \zeta \leq 1$
(2.11)

The expressions for the wetted perimeter, P, and the channel cross-sectional area, A, may respectively be expressed as

$$P = b + 2\zeta h\left(\frac{\pi\theta}{180}\right) + 2h\, cosec^2\theta \; - 2\zeta h(cosec^2\theta - cot\theta) \quad (2.12)$$

And

$$A = bh + 2\zeta h^2(cosec^2\theta - cot\theta) + h^2 cot\theta - 2\zeta^2 h^2\,(cosec^2\theta - cot\theta) + \zeta^2 h^2\left(\frac{\pi\theta}{180}\right)$$

(2.13) (Fattouh & Yousif (2020). Setting $\lim_{b\to 0} A$ and $\lim_{b\to 0} P$ makes the equations (2.12) and (2.13) reflect the situation represented by Figure 4. Subsequent optimisation based on Manning's formula and the criterion of minimal wet-perimeter yields the most efficient radius $r$ as

$$r = \frac{2^{\frac{1}{4}}\, Q_n}{\left(cot\,\theta + \frac{\theta\pi}{180}\right)^{\frac{3}{8}}\;\sqrt{S_f}} \qquad (2.14)$$

Where Q is the discharge for a given Manning's roughness coefficient, n, and $S_f$ is the longitudinal slope. Furthermore, the other optimal conditions for radius $r$, top width $W$, cross-sectional area $A$, wetted perimeter, $P$, as summarised by Experto en Ingenieria (2022), in an informative online video narration,


Received: 20-06-2023 Accepted: 20-09-2023 Published: 27-12-2023

$r$=$h$, $W = 2r cosec^2\theta$, $A = \frac{W}{4cot\theta} - \frac{r^2}{cot\theta}\left(1 - \left(\frac{\theta\pi}{180}\right) cot\theta\right)$, (2.15)

and $P = \frac{W}{cot\theta} cosec^2\theta - \frac{2r}{cot\theta}\left(1 - \left(\frac{\theta\pi}{180}\right) cot\theta\right)$ (2.16)

**Minimising the fluctuation of velocity above the free surface of the optimised channel**

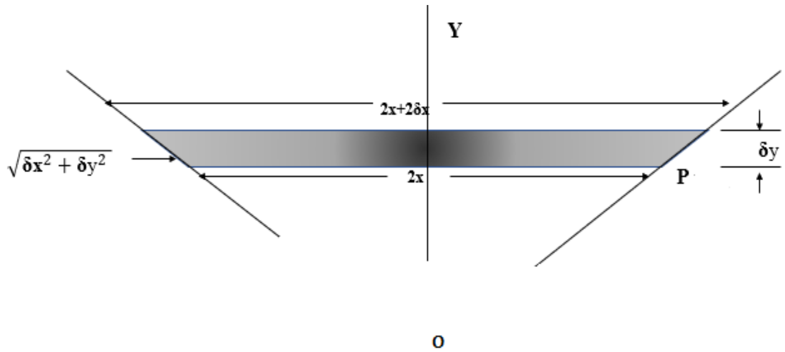


**Fig. 4: An element cross-sectional area above the open surface of the channel**

Figure 4 shows the top surface of a compound round-bottomed channel with constant hydraulic radius $m = \frac{A}{P}$, which is the only requirement according to Manning and Chezy formulae to keep velocity constant. The shaded section is a trapezoidal area element with horizontal sides equal to $2x$ and $2x + 2\delta x$ ,and a thickness $\delta y$.

The element area is $A = \frac{((2x+2\delta x)+2x)}{2}(\delta y)$= $2x\delta y + \delta x\, \delta y$, while the wetted perimeter, $P = 2\sqrt{\delta x^2 + \delta y^2}$ Therefore, the hydraulic radius $m = \lim_{n\to\delta x=0,\delta y=0} \frac{2x\,\delta y + \delta x\,\delta y}{2\sqrt{\delta x^2+\delta y^2}}$ , that is $m = \frac{x\,dy}{\sqrt{dx^2+dy^2}} \Rightarrow m^2 = \frac{x^2\,dy^2}{dx^2+dy^2}$ and $m^2(1 + \left(\frac{dy}{dx}\right)^2) = x^2\left(\frac{dy}{dx}\right)^2 \Rightarrow \int dy = \int \frac{m}{\sqrt{x-m^2}} dx$, which yields $y = m\, ln\left(x + (\sqrt{x^2 - R^2})\right) + k$ (3.1)

In view of Figure 4, where the equation of the slant side of the channel is $y = x\tan\theta$ i.e. (choosing the origin, O appropriately), the value of the arbitrary constant *k* may be determined whenever *x* is known (Rajput, 2006). The channel equation (3.1) is applicable in designing


Received: 20-06-2023 Accepted: 20-09-2023 Published: 27-12-2023

the top sections of any regularly shaped channels for minimal velocity fluctuations during overflow.

**Conclusion**

This paper has used a purely analytical approach to explore the efficiency of the hybrid round-bottom triangular open channel with minimum velocity fluctuation above the free surface. The motivation for exploring hybrid shapes is the fact that the semi-circular channel, known to rank highest in efficiency unfortunately falls short in utility due to propensity for scouring, and difficulty in layering. The optimum dimensions of the round-bottom triangular channel with minimum velocity fluctuation in stormy conditions were explored and established to be $y = m\, ln\left(x + (\sqrt{x^2 - R^2})\right) + k$ with the significance of each symbol as elaborated in section 3 above.

## References

Chahar, B., & Basu, S. (2009). Optimal design of curved bed trapezoidal canal sections. Proceedings of the Institution of Civil Engineers. *Water Management 162,* June 2009 Issue WM3, 233-240.

Douglas, J., Gasiorek, J., & Swaffield, J. (2001). *Fluid mechanics.* Prentice Hall. 182, 514-558.

Froehlich, D. (2008). Most hydraulically efficient standard lined canal sections. *Journal of Irrigation and Drainage Engineering, 10.*1061/(ASCE)0733-9437(2008)134:4(462).

Experto en Ingenieria. (2022, April). Round bottom triangle channel with the best hydraulic section by minimizing the wetted perimeter. YouTube. https://youtu.be/R9T6yCkR2Uo

Fattouh, M. & Yousif, N. (2020). Minimizing the cost of a curved corner trapezoidal canal section. *Water Supply. 21*. 10.2166/ws.2020.317.

Hameed, A.T. (2010). Optimal design of round bottomed triangle channels.*Tikrit Journal of Eng.Sciences 3* 31-43.

Marriott, M. & Uddin,S. (2009). Hydraulic design charts for elliptical pipes. Proceedings of Advances in Computing and Technology, (AC&T) The School of Computing and Technology 4th Annual Conference. University of East London, 120-125.

Massey, B. & Smith, J. (2006), Mechanics of fluids, Stanley Thornes. 182 414-456.



**Received: 20-06-2023 Accepted: 20-09-2023 Published: 27-12-2023**

Rajput, R.K. (2006). *Fluid mechanics and hydraulic machines*. S. Chand and Company, 794-795.

Wahome, J. (2014). An investigation of free surface fluid flow topologies using an inverse conformal mapping inferencing technique. Unpublished PhD thesis, University of Eldoret, Kenya.



**Received: 20-06-2023 Accepted: 20-09-2023 Published: 27-12-2023**